A new approach to modelling claims due to natural hazards

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Abstract: United Nations International Strategy for Disaster Reduction defines risk of natural disaster as “a potentially damaging phenomenon that may lead to loss of life or injury, property damage, social and economic disruption or environmental degradation”. Each hazard is characterized by location, intensity, frequency and probability. It is interesting to study inter-arrival time between two disasters in a vulnerable geographic area. In this article, a new approach to model inter-arival time between two disasters based on Stoynov distribution and process is considered.

Key words: risk; flood; Stoynov distribution; Stoynov process

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1. Introduction.

United Nations International Strategy for Disaster Reduction defines risk of natural disaster as “a potentially damaging phenomenon that may lead to loss of life or injury, property damage, social and economic disruption or environmental degradation”. Each hazard is characterized by location, intensity, frequency and probability. It is interesting to study inter-arrival time between two disasters in a vulnerable geographic area. In this article, a new approach to model inter-arival time between two disasters based on Stoynov distribution and process is studied. The approach is firstly presented in Stoynov, Zlateva and Velev (2015). In this article, the approach is further investigated

The article is organized as follows.

In Section 2, Stoynov type distribution and process are shortly presented. In section 3, applications of Stoynov process to model inter-arrival time of natural hazards are considered.

2. Stoynov distributions and Stoynov process.

We say that a random variable ξ with probability mass function $f_\xi(x)$ has distribution of $ST(n,\beta)$ family and denote this fact $\xi \in ST(n,\beta),$ if the probability mass function of ξ is given by the formula

$$f_\xi(x) = \begin{cases} \sum_{k=1}^{n+1} P(D^n = k) f_\xi(x \mid D^n = k) = \sum_{k=1}^{n+1} P(D^n = k) f_{G^k}(x), x \geq 0 \\ 0, x < 0. \end{cases}$$

where G^k are random variables with probability mass function $f_{G^k}(x) = f(k,\beta)$ and D^n are positive integer mixing random variables.

Here for G^k we may adopt different families of distribution.

In this article, the case of Stoynov family of first kind is considered where

$$G^k \in \Gamma(k,\frac{1}{\beta}) \equiv Erlang(k,\frac{1}{\beta}),$$

i. e. $\xi \mid D^n \equiv Erlang(D^n,\frac{1}{\beta}).$ Correspondingly, D^n is a random variable, taking values $k=1,\dots,(n+1)$ with probabilities $P(D^n = k) = \frac{C(n,\beta)n!}{\beta^k (n-k+1)!}, k=1,\dots,(n+1)$ where the coefficients $C(n,\beta)$ are given by the formulas:

$$C(n,\beta)=\frac{1}{I(n,\beta)},$$
$$I(0,\beta)=\frac{1}{\beta},$$
$$I(n,\beta)=\frac{1}{\beta}+\frac{n}{\beta}I(n-1,\beta), n=1,2,\dots$$

Also, variables $\tilde{D}^n = D^n - 1$ can be introduced taking values $k=0,\dots,n$ with probabilities

$$P(\tilde{D}^n = k)=\frac{C(n,\beta)n!}{\beta^{k+1}(n-k)!}, k=0,\dots,n.$$

Then the probability mass function $f_\xi(x)$ of ξ may be presented also by the formula

$$f_\xi(x)=\begin{cases}\sum_{k=0}^{n} P(\tilde{D}^n = k)\, f_\xi(x \mid \tilde{D}^n = k) = \sum_{k=0}^{n} P(\tilde{D}^n = k)\, f_{G^{k+1}}(x), x \ge 0\\ 0, x<0.\end{cases}$$

If a random variable has Stoynov distribution of first kind, we will denote this fact $\xi \in ST1(n,\beta)$.

If in $ST1(n,\beta)$ we enforce $D^n = k$, i. e.

$$P(D^n = k)=1, P(D^n = i)=0, 1\le i\le k-1<k+1\le i\le n+1,$$

which may be considered as degenerate Stoynov distribution of first kind, we actually obtain Erlang distribution, i. e. $\xi \in Erlang(k,\frac{1}{\beta})$.

$ST1(n,\beta)$ distribution can be considered as a special kind of generalized gamma distribution (Stoynov, 2011).

We say that a random variable Λ has generalized gamma distribution if its probability density function is given by

$$f_\Lambda(x)=\frac{u^{s-\alpha}}{\Gamma(\alpha)U(\alpha,\alpha+1-s,u\beta)}e^{-x\beta}x^{\alpha-1}(u+x)^{-s}, x>0$$

where $-\infty<s<+\infty, \alpha>0, \beta>0, u>0$ and

$$U(a,b,z)=\frac{1}{\Gamma(a)}\int_0^\infty e^{-zt}t^{a-1}(1+t)^{b-a-1}dt, a>0, z>0$$

is the integral representation of the hyper-geometric function of second kind (Zamani and Ismail, 2010). In this case, for Λ we write $\Lambda \in G\Gamma(\alpha,\beta,u,s)$.

To remember that we say that the random variable ξ has a gamma distribution and denote $\xi \in \Gamma(\alpha,\beta)$, if its probability mass function is given by

$$f_\xi(x)=\begin{cases}\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}, x\ge 0\\ 0, x<0\end{cases}.$$

Here $\Gamma(\alpha)$ is defined by $\Gamma(\alpha)=\int_0^\infty x^{\alpha-1}e^{-x}dx.$

We see that $\Gamma(\alpha,\beta)=G\Gamma(\alpha,\beta,u,s=0)$.

Actually, we have that

$$ST1(n,\beta)=G\Gamma(\alpha=1,\beta,u=1,s=-n).$$

The exponential distribution is a special kind of $ST1(n,\beta)$ distribution are $ST1(0,\beta)\equiv Exp(\beta)$. To recall that the random variable ξ has exponential distribution and denote $\xi \in Exp(\beta)$, if its probability mass function is given by

$$f_{\xi}(x)=\begin{cases}\beta e^{-\beta x},\ x\geq 0\\ 0,\ x<0.\end{cases}$$

The case $ST1(1,\beta)\equiv Lindley(\beta)$. can be considered as a weighted version of exponential distribution with weighting function $w(x)=1+x$. We say that the random variable ξ has Lindley distribution and denote $\xi \in Lin(\beta)$, if its probability mass function is given by

$$f_{\xi}(x)=\begin{cases}\dfrac{\beta^2}{\beta+1}(1+x)e^{-\beta x},\ x\geq 0\\ 0,\ x<0\end{cases}.$$

The case $n=2$ leads to $ST1(2,\beta)$ distribution which can be defined as weighted exponential distribution by the weight function $w(x)=(1+x)^2$.

We say that the random variable ξ has $ST1(2,\beta)$ distribution and denote $\xi \in ST1(2,\beta)$, if its probability mass function is given by

$$f_{\xi}(x)=\begin{cases}\dfrac{\beta^3}{\beta^2+2\beta+2}e^{-\beta x}(1+x)^2,\ x\geq 0\\ 0,\ x<0.\end{cases}$$

To check that the weight function in the definition of the distribution is the right one, we could calculate $Ew(\xi)=\dfrac{\beta^2+2\beta+2}{\beta^2}$.

We say that a process $X(t)$ is a Stoynov process of first kind or $ST1(n,\beta)$ process, and denote this fact $X(t)\in ST1(t;n,\beta)$, if for it:

1) $X(0)=0$.
2) $X(t)$ is pure jump process with jumps at times $T_{i'}, i=1,2,\ldots$ and jump sizes

$$\Delta X(T_i)=1.$$

3) The intervals between two jumps are

$$\tau_i=T_i-T_{i-1}\in ST1(n-1,\beta), i=0,1,\ldots,T_0=0.$$

We say that $X(t)$ is compound $ST1(n,\beta)$ process if we replace condition 3) with condition:

3') $X(t)$ is pure jump process with jumps at times $T_{i'}, i=1,2,\ldots$ and jump sizes $\Delta X(T_i)=Y_i$, where Y_i are independent and identically distributed random variables.

3. Applications of Stoynov process to model inter-arrival time of natural hazards.

In Stoynov, Zlateva and Velev (2015), example with modeling inter-arrival time between floods by Stoynov processes is presented.

4. Conclusion.

Stoynov distributions presented in the article posses some suitable properties for modeling process of arrival of floods.

The present work may be extended for typical (not degenerated) ST processes as well as by studying other choices of G^k.

For example, we may choose $G^k \in NB(k,e^{-\beta})$. In this case, $\xi \mid D^n \equiv NB(D^n,e^{-\beta})$ and we obtain a distribution which we will call $ST2(n,\beta)$ distribution.

As another example we may consider $G^k \equiv \delta_k(x)$ - random variable which takes value k with probability one. Then $\xi \equiv D^n \in D^n(\beta)$. In this case, we say that random variable ξ has $ST3(n,\beta)$ distribution and denote $\xi \in ST3(n,\beta)$.

These processes with suitable parameters can also be used to model times of occurrence of floods.

Further step is to test the model against real data and to check other proposals for the first parameter of Stoynov distribution and process.

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