Fifteenth International Conference
Financial and Actuarial Mathematics – FAM-2022

An entire study about the development of newer methods for transportation problem optimization

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Abstract.
The transportation problem is a widely use optimization technique to analyze the problems concerned with transporting goods.The transportation sector has become eminent based on the economic, social factors and also it influences the relationships existing among various countries. With the movement of goods and people, the transportation sector promotes the travel and tourism sector. The transportation problem is one among the original endeavors of a linear programming model. It is an optimization problem having linear constraints and linear objectives.The main goal of the transportation problem is minimization of transportation cost, minimization of distance with respect to time and so on. There are several optimization techniques developed in recent years, to solve the transportation problem.This explorative paper explores about the transportation problems and the recent advancements in the optimization methods developed for decoding the transportation problems. The proposed exploration provides insights about the methods used for finding initial basic feasible solution to a transportation problem and the newly developed methods based on these techniques. There are many alternative methods for providing solutions to the transportation problems. For any transportation problem, the usual methods, which are employed to find the initial basic feasible solution is the Vogel's Approximation Method (VAM), the Least Cost Method (LCM) and the North-West Corner method. This research helps in researching further about the developing cost-effective methods that assists in minimizing the transportation costs.

Keywords: Transportation cost, transportation problem, feasible solution, optimal solution etc.

Introduction

The transportation sector forms an eminent part of the human beings. The movement of things is essential to be done and they are achieved with the help of transportation systems. These transportation systems possess high efficiency, speed and development over time with a drastic enhancement (impoff, 2020). The eminence of transportation is highlighted in how the governments, businesses and individuals rely on this sector for accessing the resources. An optimal functionality cannot be contributed by a society, when it lacks transportation facilities. A sustainable development and an enhanced total productivity is contributed by the transportation sector. Economies, job, resources and people require indefectible movement for retaining the further advancement of the whole world. The transportation problem is one among the original endeavors of a linear programming model. It is an optimization problem having linear constraints and linear objectives. Lots of special structures are exhibited by the transportation problem. For instance, each of the variables occurs in accurately with two non-zero coefficients. The variable encompassing this coefficient possesses either negative or positive value. There are many alternative methods for providing solutions to the transportation problems. For any transportation problem, the usual methods, which are employed to find the initial basic feasible solution is the Vogel's Approximation Method (VAM), the Least Cost Method (LCM) and the North-West Corner method. For a transportation problem, the optimality is checked with the MODI method. The newer methods for transportation problems' optimization is designed with the alterations of the above mentioned methods.

Theoretical background of the study

The theoretical concepts and terms of the proposed research are discussed here.

Operations Research

Operations research is defined as the application of quantitative methods for helping analysts and decision-makers for enhancing, analyzing and designing the operation or performance of the systems. The operations research field encapsulates analytical tools from various fields that could be embedded in an efficient way for assisting the decision-makers in resolving various problems in an advantageous or practical way. A system's performance could be optimized with the tools of operations research. Probability and statistics, optimization and simulation are eminent features of an operations research. Identification of a problem, construction of a model having variables in the real-world; usage of this model for deriving resolutions for a problem and testing and analyzing the solutions of the implemented model are eminent steps of operations research. Some endeavors of operations research are; risk management, packet routing optimization, inventory management, time management and scheduling, agricultural and urban planning, engineering and network optimization, supply chain management and in enterprise resource planning.

Linear programming problem

A linear programming problem encompass finding of a minimized or maximized subject involving a linear function to some constraints in linear inequalities or equalities form. This programming model is employed for gaining the greater optimum solution for a constraints carried transportation problem. Some of the types of a linear programming model are; linear programming model with an open solver, linear programming with graphical method, linear programming by R method, linear programming by simplex method. The components of a linear programming model are; objective functions, decision variables, data and constraints. Non-negativity, finiteness, linearity are some features of linear programming. Some of the linear programming applications are; transportation optimization, energy industry, efficient manufacturing and engineering.

Transportation problem

A transportation problem is one among the forms of a linear programming problem, which involves the transportation of services and goods to various destinations from various sources. This is in accordance with the demand and supply of the suitable destinations and sources with the lowest minimum cost of transportation. The main objective of a transportation problem is the minimization of the transportation cost.

Mathematical representation of transportation problems

A transportation problem is concerned with the movement of products from m number of supply sources to n number of distribution units or destinations. Minimization of the total transportation cost is the main objective. O is the supply source and D is the destination. Hence goods are transported from O_1, O_2, O_3……O_m to D_1, D_2, D_3…….D_n. The supply units are indicated by a_i and the demand units are indicated by b_j and c_{ij} is the cost of transportation per unit from O to D. here i= 1,2, 3, 4,…., m and j =1, 2, 3, 4,….,n. For fulfilling the demand and supply needs with the low transportation cost, the associated linear model is expressed as follows;

$$\text{Minimum } z= \sum_{i=1}^{m}\sum_{j=1}^{n} xijcij$$

Which is in accordance with the constraints, $\sum_{i=1}^{m} xij \geq bijand \sum_{j=1}^{n} xij \leq aij$ and $x_{ij} \geq 0$ for i and j values. Thus, these are the standard mathematical formulations of a transportation problem.

Initial basic feasible solution

A group of non-negative allocations, where there is a satisfaction of columns and row restrictions is referred to as a feasible solution. A feasible solution for origin m and destination n is regarded as a basic feasible solution where the number of optimistic allocations is m+n=1.

Optimum solution

The optimum solution is a type of feasible solution, which involves the reaching of the objective function to minimum or maximum value; for instance, the highest least cost or the profit. When there are no feasible solutions having functional values, then it is called a global optimum solution.

Balanced transportation problem and unbalanced transportation problem

When the total supply units' number equals with the total demand units' number, then it is called a balanced transportation problem. When the supply units' number and the demand units' number are unequal, then it is an unbalanced transportation problem.

Literature Review

(Wagener 1965) developed a new model for providing solutions to the transportation problems. There a new procedure was introduced for the transportation model computation, where every column in the cost matrix is utilized and thus the method was suitable for the computers having tape storage. However, some limitations and errors were identified in his proposed model, which lead to further research on the field.

(Ekanayake et.al 2020) developed a modified ant colony optimization technique for getting transportation problem solution. A modified ant colony optimization technique was proposed by them with a meta-heuristic algorithm for getting the initial basic feasible solution for transportation problems. The model possessed finite iterations and was simple in execution. The accuracy of the algorithm was made with numerical examples and it offered satisfactory results. With their new algorithm, the time consumption of the transportation problems is reduced.

(BilqisAmaliaha, ChastineFatichaha and Erma Suryanib 2020) constructed a newer heuristic method for determining the initial basic feasible solution for a transportation problem. This new method was indicated as Bilqis Chastine Erma (BCE) method. 35 numerical archetypes were utilized for evaluating their new method's performance. The experimental results of the study made a conclusion that this newer method provided high efficiency of 88% when compared with the other transportation methods.

(M.S.R Shaikh, S. F. Shah and Z. Memon 2018) constructed an enhanced algorithm for getting initial basic feasible solution for a transportation problem. The new method was tested for accuracy against the North-West Corner method and the Least Cost methods, utilizing five numerical archetypes. It was concluded that much reliable results were got with the new method, when compared to the other methods.

There are other new methods developed and designed with some alterations made to the North West corner method, least cost method and Vogel approximation method.

Methodology

The traditional optimization techniques were helpful in determining the unconstrained minima or maxima of differentiable and continuous functions or the optimum solution. The above are analytical techniques and they utilize different calculus for optimum solution location. Some of the numerical optimization techniques are, combinational optimization, constraint satisfaction, dynamic programming, stochastic programming, non-linear programming, quadratic programming, integer programming, linear programming and dynamic programming. Some of the advanced optimization methods are, hill climbing, simulated annealing, genetic algorithm, ant colony optimization etc (nptel, 2020). Among the other eminent optimization techniques, the north-west corner rule, Vogel approximation method and least cost method are important techniques of optimization that helps in finding the initial basic feasible solution for a transportation problem.

North West Corner Method

In the North-west corner method, focus must be made on the North-west comes of the transportation table to find out the initial basic feasible solution. There are three important steps in a North-west corner rule which are as follows:

- Determine the cell with the North-west corner of the transportation table. Allocation must be possibly made to the chosen cell with the respective adjustment of the related quantities of demand and supply by extracting the amount allocated.
- Then the column or row that has zero demand for supply must be crossed out. When both a column or a row posses a o, then that column or row must be randomly crossed out .The following shows an empty transportation table having supplies in the right side and at the bottom the demands as shown in the below table.

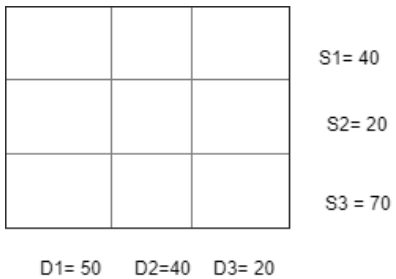

Figure 1: An empty transportation table with demand and supplies

Here, we initially choose the cell on the North-West Corner and its associated value is a minimum of S1 and D1. When D1 is greater than S1, then the first row must be crossed out. This step must be repeated until we get a table with the basic feasible solution.

New methods developed out of North West Corner Method

(Metlo et.al 2016) made a modification in the North-West Corner method for making optimization of the transportation problems. This newer method is modified forms of the North West Corner Method to get the initial basic feasible solution along with culminate entries required for operating the transportation that is utilized for obtaining the optimal solution. In linear programming, the transportation problem is an eminent model class

and is associated with the transportation of commodities to multiple destinations with the purpose of reducing the transportation steps or time. The aim of their modification was to search for the cheapest and best way, the suppliers are utilizing for satisfying the demand at the eminent points. There would be an adjustment from the distributor to the points of demand during a change in the shipping products cost. Then a comparison was made between the North West Corner method and the modified North West Corner method. The comparison results were equal and the newly modified method was greater in power when compared with the conventional North West method.

(Neetu Sharma and Ashok Bhadane, 2016) developed an alternative model of the North-west corner method to solve the transportation problem. Their new models is developed by the help of a statistical tool, which is referred to as co efficient of range. Numerical archetypes were explained in their paper for justifying their results. These numerical archetypes illustrated that the entire transportation cost, which is got by the utilization of the coefficient of range (COR) provides optimal results when compared with the North-west corner method. In addition, the transportation cost which is got with the new COR method was similar to the least cost method. A non-degenerate solution was obtained with the North-west corner method. It showed that the initial basic feasible solution got with the COR method is very less. When compared with the iterations number.

Least Cost Method (LCM)

Least cost method (LCM) is one among the methods, which is used for getting the initial basic feasible solution in a transportation problem. Here, the cell having the minimum transportation cost is allocated first. These cells of low cost are selected over the cells of high cost with a view for getting the minimum transportation cost or least transportation cost. More solutions for transportation with optimality are generated by the Least Cost method, when compared against the North-West Corner method, as it takes into account the cost of shipping during the allocation process. Only the supply needs and availability are considered in the North-West Corner method, which is independent of the shipping cost. The demand and supply must be equal and when there is more supply, there is an addition of a dummy source in the transportation table and the demand value equals the demand and supply difference value and zero is the cost. When the demand is higher than the supply, there is an addition of a dummy origin or destination to the transportation table and the supply value is equal to the difference between demand and supply and zero is the cost value. The following shows the steps of optimization of the transportation problems using the least cost method (LCM).

- Initially we have to check whether a given transportation problem is balanced or not. When the supply is not equal to demand or in an unbalanced condition, a dummy market or source is added to the transportation table.
- Choose the lowest transportation cost out of the total transportation matrix and make an allocation of the minimum demand or supply.
- Then the column or row that has a fulfilled demand or supply must be removed and then a new matrix must be prepared.
- The above procedure must be repeated until the end of all allocations.

New methods developed out of LCM Method

(Syed Nasir Mehmood Shah et.al 2010) made a modification of the least cost method for resource allocation in the grid. The criteria for sharing resources are the grid. Any service quality is influenced by proper grid management. In their research, they developed a new method for resource allocation in the grid. That method was a modified least cost method, which prepares tasks for computing optimum nodes. That was an enhanced technique of the least cost method. They evaluated that method for the scenario of resource allocation in the grid. In addition, for grid scheduling, a linear programming model was proposed, which encapsulated a comparison of the proposed method with that of the available methods. Satisfactory results were obtained with their proposed technique.

(M. Kavitha and N. Srinivasan 2018) proposed another new method in place of the least cost method for the optimization of the transportation problem. That method is otherwise known as the Lowest Even Cost method. The lowest even cost method was developed for finding the initial basic feasible solution for a given problem in transportation. Their method was found to be a complex method for calculation, when compared with the other methods. A better initial basic feasible solution was gained with the method, but the accuracy of the method was found to be unsure.

Vogel's Approximation Method (VAM)

Vogel's Approximation Method is one among the technique used for determining the first basic feasible solution for a given transportation problem. The steps for the optimization of a transportation problem using Vogel Approximation Method are given below.

▪ The technique initiates with the determination of the two minimum transportation cost cells in each column and row in the array of the transportation problem. The minimum of the transportation costs is subtracted from each other, which generates a Vogel number for each column and row.

▪ Choose the highest Vogel number and then assume in the respective minimum cost cell, where the assumption must be highest amount, which is transferred to the respective destination. Post this assignment, the recompilation of the Vogel numbers is made, contemplated on the existing columns and rows of the transportation array. The entire process is repeated till the entire shipments are done.

Computer based software is essential for the extra work of computation utilizing the Vogel approximation Method in offering transportation problems' solutions.

New methods developed out of VAM method

(Pratihar et.al 2020) made a modification of the Vogel's approximation method for solving the transportation problems in an uncertain environment. In their research, the interval type 2 fuzzy set was embedded in the fuzzy set transportation problem for highlighting the supply, transportation cost and demand. The fuzzy sets and their types embedded in the transportation problems are discussed in this research. The Vogel's approximation method was modified to provide solution to the transportation problem of fuzzy set. Numerical examples were discussed for describing the system accuracy. Their research insists future analysis for application of their proposed technique in a practical transportation problem on a large scale.

(Daniel Shimshak et.al 2016) developed an alternative form of Vogel's approximation method with the application of heuristics were utilized with the Vogel Approximation Method (VAM) in their proposed technique. Computations were made with test problems, which highlighted the accuracy of the heuristics. The method was found to consume less effort and time needed for obtaining an initial basic feasible solution utilizing Vogel approximation method. This result was found by both manual and computational operation.

Conclusion

Thus, this paper of exploration initially explores about the theoretical concepts associated with the transportation problems of operations research, then a disquisition review about the literature existing for the proposed research, the methods used to solve the transportation problems and the review on the recently developed advanced methods for optimizing the problems in transportation. Transportation problems were earlier resolved utilizing simplex method. But it was a time-consuming process and produced many errors. Artificial variables are ignored and main variables are only considered in a transportation problem. Thus, the above-mentioned linear programming models were developed for resolving both transportation and simplex problems. Transportation cost is an eminent parameter for optimization of problems in transportation. The new transportation methods are developed based on this objective.

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