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Sequences of Topological Spaces: New Results

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Abstract: Within the concept of the series, we deal with the subject of calculus. When studying the series, we explore the limits of series. Also in topology we will refer to series of real numbers Abd Algani (2019); Dirac (1961); Kozov (2008) proved some results in the sequences of topological spaces. In the present study we will begin by defining the series in Group X, and present the following claims:

Lemma 1: (X, d) will be a metric space. The series (x_n) converges to the point $a \in X$ if and only if for every $\epsilon > 0$ there is a natural number N so that $d(x_n, a) < \epsilon$ for every $> N$.

Lemma 2: (X, T) will be a topological space and $A \subseteq X$. If a is a limit point for a series within group A, then a belongs to the closed group A.

Lemma 3: f will be a continuous function from the topological space (X, T) to the topological space (Y, S) and (x_n) will be a series in X that converge to a, then the series $\left(f(x_n)\right)$ will converge to $f(a)$.

Key words: Topological space, Metric space, Distance.

Introduction

Let's start with defining the function of distance:

Definition 1. Let R be a set of real numbers and a, b will be two elements in group R. We will define the distance $d(a, b)$ as follows:

$$d(a, b) = |a - b|.$$

So, $d: R \to R$ meets the following four conditions for every $a, b, c \in R$:

1. Non-negativity. $d(a, b) \geq 0$
2. $d(a, b) = 0$ if and only if $a = b$
3. Symmetries of $d(a, b) = d(b, a)$.
4. $d(a, c) \leq d(a, b) + d(b, c)$ (triangle inequality)

Definition 2. Let A_1 be a partial group of the metric space (A, d), and b will be an element of group A. The point b is called the limit point of group A_1 if each environment N of the point b contains at least one point of the points A_1 which is different from b, that is,

$$N \cap (A_1 - \{b\}) \neq \emptyset$$

From the above definition it can be concluded:

If b is a limit point for group A_1 then there is a series of points from A_1 that aspires to b. But if $b \in A_1$ so that there is an environment N to the point b and $A_1 \cap N = \{b\}$, then we call the point b, the isolated point from A_1.

Example 1: Let R will be the set of real numbers, and the metric is the matric of the absolute value and $A_1 = (0,1]$.

So the 0 is one of the limit points for group $A_1$and $\left\{1, \frac{1}{2}, \frac{1}{3}, \dots\right\}$ is a series that converge to 0.

But if the group is $A_1 = \{1, 2, 3, \dots\}$ then all the points are isolated if we assumed that $n \in A_1$ then $\left(n - \frac{1}{2}, n + \frac{1}{2}\right)$ is a surrounding for the point n and

$$\left(n - \frac{1}{2}, n + \frac{1}{2}\right) \cap A_1 = \{n\}$$

Definition 3. The function f from the set of natural numbers N to the set X is called a series in X.

It is clear that the function f is a defined group. Here, we will focus on the range of the function and we will mark it in $f(n) = x_n$ so that $x_n$is an element of X and we will mark the group with (x_n).

Definition 4. Let (X, T) be a topological space and (x_n) a series of elements from X. Suppose that (x_n) is a function that aspires to a , so that $a \in X$ if and only if for each partial open set A of X contains a. There exists a natural number N so that $x_n \in A$ for each $n > N$. In other words, a contains all the elements of the series except for a finite number of elements.

New results

Lemma 1. (X, d) will be a metric space. The series (x_n) converges to the point $a \in X$ if and only if for every $\epsilon > 0$ there is a natural number N so that $d(x_n, a) < \epsilon$ for every $n > N$.

Proof of Lemma 1: Suppose the series (x_n) aspires to point a, and suppose there exists $\epsilon > 0$, then the open ball $B(a, \epsilon)$ represents an open environment group to the point a, then $B(a, \epsilon)$ contains the elements of the series (x_n). That is, there exists a natural number N so that x_n belongs to $B(a, \epsilon)$ per $n > N$. That is $d(x_n, a) < \epsilon$ per $n > N$. This means that the series converges to a.

Lemma 2. (X, T) will be a topological space and $A \subseteq X$. If a is a limit point for a series within group A, then a belongs to closed group A.

Proof of Lemma 2: x_n will be a series converging to a, so that $x_n \in A$ per N.

Suppose U is an open group containing a. That is U contains all the elements of the series (x_n) except for a finite number of elements, then $U \cap A \neq \emptyset$

This leads to a point in a closed group A.

Lemma 3. f will be a continuous function from the topological space (X, T) to the topological space (Y, S) and let (x_n) be a series in X that converge to a, then the series $\left(f(x_n)\right)$ converge to $f(a)$.

Proof of Lemma 3: B will be an open group from Y, which contains the point $f(a)$ then $f^{-1}(B)$is an open group from X that contains the point a.

That is $f^{-1}(B)$ contains all the elements of the series (x_n) except for a finite number of elements. This implies that B contains all the elements of the series $\left(f(x_n)\right)$ except a finite number of elements,

Then the series $f\left(f(x_n)\right)$ converges to point $f(a)$.

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